Supply Chain Fundamentals: Inventory Management

Notes from “Supply Chain & Logistics Fundamentals”

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# Inventory Concepts & Definitions

Inventory is held for 5 reasons:

|  |  |
| --- | --- |
| Cover process time | If ordered in batches, inventory will be kept on hand to meet demand over time. |
| Allow for uncoupling of processes | Buffer stock held between stages in a multi-stage process. This keeps one process from having to wait on the other. It also allows someone to perform all of the work in the first stage before moving to the second, rather than switching between stages/tasks for each unit produced. |
| Anticipation / Speculation | Building up extra stock for anticipated periods of increased demand. |
| Minimize control costs | ? |
| Buffer against uncertainties | Maintain high service levels when there is uncertainty in demand, supply, delivery, or manufacturing / processing. |

Three levels of inventory decisions:

|  |  |
| --- | --- |
| Strategic | Supply Chain Decisions   * What are the potential alternatives to inventory? * How should product be designed? |
| Tactical | Deployment Decisions   * What items should be carried as inventory? * In what form should they be maintained? * How much of each should be held and where? |
| Operational | Replenishment Decisions   * How often should inventory status be determined? * When should a replenishment decision be made? * How large should the replenishment be? |

For our purposes, inventory is classified based on its function (as opposed to other ways to classify inventory, such as for accounting purposes). The functional roles of inventory are:

|  |  |
| --- | --- |
| Cycle Stock | Inventory used to meet demand between replenishment times |
| Safety Stock | Additional stock used as a buffer for uncertainty |
| Pipeline Inventory | Inventory that has been ordered but not yet delivered. It is “in the pipeline.” Also called “Inventory On Order.” |

TODO: better picture (show pipeline?)

We can also measure inventory in different ways:

Inventory Position (IP) = Inventory On Hand (IOH) + Inventory On Order (IOO) – Backorders

* Inventory Position = total inventory in system (includes on hand and on order, minus backorders or committed inventory)
* Inventory on Hand = physical inventory on hand
* Inventory On Order = inventory that has been ordered but not yet delivered
* Backorders = product that has been ordered but for which the customer is willing to wait. We will still need to satisfy this demand out of future inventory.

These notes will describe various inventory management techniques and their impact on inventory and service levels. There are several different inventory and ordering models, each using different assumptions. In each case, an optimal policy is sought that will minimize costs and maximize profits.

When we examine total costs of an inventory management model, we will typically break this down into four components:

Total Cost = Purchase + Ordering + Holding + Shortage

* Purchase (Unit Value) Cost – variable cost (per unit) to purchase inventory
* Ordering (Set Up) Cost – cost to place an order
* Holding (Carrying) Cost – cost to carry inventory
* Shortage Cost – cost of not meeting demand (being “short” or out-of-stock)

# Variables & Notation

|  |  |  |  |
| --- | --- | --- | --- |
| Symbol | Name | Description | Units |
| D | Total Demand (over period of time being examined) | Customer demand for an item. This demand must be met or there will be a shortage. | units / time |
| c | Variable (purchase) cost | Cost of purchasing 1 unit | $ / unit |
| ct | Fixed Order (transaction) Cost | Cost of placing an order | $ / order |
| ce = c ∙ h | Excess Holding Cost | Cost of holding excess inventory | $ / unit / time |
| h | Carrying or Holding Charge | Excess holding cost (expressed as a percentage of unit cost) | $ / unit / time  (% of unit cost) |
| cs | Cost of a shortage | Cost of being short and not meeting demand | $ / units / time |
| L | Lead Time | Time between when an order is placed and when it is delivered | time |
| Q | Replenishment Order Quantity | Quantity ordered | units / order |
| T | Order Cycle Time | Time between order points | time / order |
| N = 1/T | Orders per Time | Average number of orders placed in a given time period | order / time |
| TC | Total Cost | Total cost of an inventory management policy | $ / time |
| TRC | Total Relevant Cost | The portion of Total Cost that can be influenced by one more variables under our control. This is different for each model. | $ / time |

CONSIDER: moving Inventory Position and Total Cost notes down here

# Economic Order Quantity (EOQ)

The “Economic Order Quantity” calculates an optimal ordering policy under a set of simple assumptions. The solution balances the costs of ordering with the cost of holding inventory and is very robust with respect to how these costs are estimated. This model can be extended in several ways and provides a good starting point for analyzing inventory and order quantities.

|  |  |  |  |
| --- | --- | --- | --- |
| Q  T  D | Assumptions:   * Constant demand * No shortages * Instantaneous delivery * Unlimited capacity * No perishability * Infinite planning horizon   Optimal Policy:   * Order Q\* Units Every T\* time periods * Order Q\* Units When IOH = 0  |  |  | | --- | --- | |  |  | |

*Additional Outputs:*

|  |  |
| --- | --- |
|  | (Q = DT so that Qty Ordered = Qty Sold) |
|  | (area of each triangle in “Inventory” graph above) |

*Costs:*

*Optimal Solution:*

|  |  |
| --- | --- |
|  | NOTE: This is found by taking the derivative of TRC(Q) and setting it equal to zero. |
|  |  |

The optimal solution is very robust with respect to how it is calculated and even how it is implemented. If we estimate demand incorrectly (forecasting DF when it is actually DA) we will see total relevant costs change by:

|  |  |
| --- | --- |
| Notice that it is better to over-forecast than to under-forecast and that even if we over-forecast by 50% the solution is only 2% worse than optimal. If we under-forecast by 50% the solution will be 6% worse. |  |

Estimates for ordering costs (ct) and inventory costs (ce) behave similarly to those shown above for demand (D) since they are all part of the same term. The only difference is that it is better to under-estimate inventory costs rather than over-estimate them.

If we estimate the costs correctly but implement a different policy: ordering with Q rather than Q\*, Total Relevant Cost will be given by:

|  |  |
| --- | --- |
| Notice that it is better to over-order than to under-order and that even if we over-order by 50% the solution is only 8% worse than optimal. If we under-order by 50% the solution will be 25% worse. |  |

Estimates for cycle time (T) behave similarly to those shown above for order quantity (Q).

*Cycles Times and the “Power of Two” Rule*

If you are ordering multiple items, it doesn’t make sense to calculate the optimal cycle time for each one and order them at different times. Instead, it is usually better to order them together. The “Power of Two” rule is used to provide a range of times that are near-optimal for each item. You can then look for overlaps between items to determine a combined order policy.

If the optimal cycle time is given by T\*, the power of two rule guarantees that you will be within 6% of optimal if you choose a value of T within the range:

If T\* is 5.1 days this gives a range of . The lecture actually said that you should choose T equal to a power of 2 ( where *k* is an integer), but I’m not sure why.

# EOQ with Non-Zero Lead Time

|  |  |  |  |
| --- | --- | --- | --- |
| Q  T  D  L | Changes to Assumptions:   * ~~Instantaneous delivery~~ * Lead Time = L   Optimal Policy:   * Order Q\* Units Every T\* time periods * Order Q\* Units When  |  |  | | --- | --- | |  |  |   (Same as normal EOQ. Only the re-order point changes) |

*Additional Outputs:*

*Costs:*

NOTE: Total relevant costs are the same as before (since the “+DL” term doesn’t involve Q and thus is fixed). This is why the optimal solution, Q\*, is the same.

# Other Extensions

Additional extensions to the EOQ model are discussed including cases where purchase cost discounts are offered (including all-item, incremental, and one-time buy discounts). Finite models are also discussed where orders arrive at a fixed rate rather than all at once (as if the item is manufactured by some process). TODO: detail these… maybe

# EOQ with Planned Backorders

|  |  |
| --- | --- |
| Q  T  D  b  T1  T2 | Changes to Assumptions:   * ~~No shortages~~ * Shortages are back-ordered with cost/unit/time = cs   Optimal Policy:   * Order Q\* Units Every T\* time periods * Order Q\* Units When back-orders = b |

*Costs*

*Optimal Solution*

NOTE: This can be found by taking the partial derivatives of , setting both of them equal to 0 and then solving for Q and b.

where:

“CR” is the “critical ratio,” and it expresses cost of shortages as a percentage of total inventory costs per unit time. If this is low (shortages do not cost much), the optimal policy will be to delay ordering, accumulating shortages for as long as possible before placing one big order (bigger than Q\*) to catch up. If this is high (shortages cost much more than excess inventory) the optimal order policy will be close to Q\* and will not allow shortages. The optimal solution is trying to find a balance between excess inventory and shortage costs.

# Single Period Inventory Model

The single period inventory model assumes that we have random demand, but we can only place one order beforehand to meet that demand. This is the classic “newspaper problem” where today’s newspaper must be produced in the morning, and at the end of the day must be thrown away (since nobody wants a newspaper from yesterday).

If we buy quantity *Q* at cost *c* and sell it at price *p*, our profit is:

*x* is the quantity demanded, and in this case it is a random variable. There are two ways to solve this:

1. We can build a data table, plotting multiple values of Q and x, calculating expected profits for each Q, and then selecting the Q that yields the maximum profit (this is especially useful when *x* is a discrete random variable).
2. We can solve analytically using marginal analysis.

Marginal analysis can be used for any continuous distribution of x. It involves an examination of:

|  |  |
| --- | --- |
|  | Expected excess cost of Qth unit ordered |
|  | Expected shortage cost of the Qth unit ordered |

As long as the expected excess cost is less than the expected shortage cost, we should increase Q, so that the maximum profit occurs when:

The critical ratio gives the cumulative probability of demand that we must satisfy. This is true regardless of the distribution of *x*. We can then use the inverse cumulative probability function to determine the value of *Q* that covers this demand:

In the simplest case we will just have:

(lost profit if short)

(cost of product, when in excess)

But we can also extend this problem to include salvage values and additional shortage penalties:

|  |  |  |
| --- | --- | --- |
| Variable | Name | Units |
| 𝑔 | Salvage value | $ / units |
| *B* | Penalty for not satisfying demand | $ / units |

Then we will need to have:

(lost profit and additional penalty for not satisfying demand)

(cost of product, when in excess and sold at salvage value, g)

*Expected Values*

It can be useful to examine the expected values for demand, sales, and units short. These are defined for continuous and discrete distributions as:

|  |  |  |
| --- | --- | --- |
|  | Continuous | Discrete |
|  |  |  |
| E[Demand] |  |  |
| E[UnitsSold] |  |  |
| E[UnitsShort] |  |  |

Notice that in each case:

This is a useful relation since we usually have E[Demand]. This means we only need to calculate either E[UnitsSold] or E[UnitsShort], and then we can use this to infer the value of the other.

We can also calculate expected profit. If we go back to the case without salvage costs or shortage penalties, we have:

With salvage costs and shortage penalties, we have:

*Solution if Demand is Normally Distributed*

For the normal distribution we have:

G(k) is the “Unit Normal Loss Function.” It provides the expected units short using a unit normal distribution. This can be calculated using a lookup table or in Excel as:

G(k) = NORMDIST(k,0,1,0) - k\*(1 - NORMSDIST(k))

It is also easy to calculate the optimal solution, since this is simply the point where the cumulative demand met equals the critical ratio:

In Excel this is:

**Base Stock Policy**

The base stock inventory policy sets a base stock level (S) and orders up to that level. Any time a unit is sold another one is re-ordered. The effect of this policy is that the Inventory Position (or base stock) remains constant although Inventory on Hand will fluctuate.



To use the base stock policy, we first must determine the required Level of Service (LOS). The base stock level will then be set to meet this LOS. LOS can be set either arbitrarily (i.e. management says we must be in-stock 95% of the time) or using the critical ratio:

where is the quantity demanded over lead time (L). If demand is normally distributed (), we can calculate the k-value corresponding to the desired level of service and then set S\* to:

# Continuous Review Policies

A continuous review policy means that we are constantly measuring inventory and we will place orders whenever our inventory reaches a certain level. There are two common types of continuous review policies:

|  |  |
| --- | --- |
| Order-Point, Order-Quantity (s,Q)   * Policy: Order Q if IP <= s * “Two-bin system” | This will order a fixed quantity (*Q*) whenever inventory reaches the re-order point *s*. |
| Order-Point, Order-Up-To-Level (s,S)   * Policy: Order (S-IP) if IP<=s * “Min-max system” | Orders up to target level *S* whenever inventory reaches re-order point *s*. Instead of a fixed quantity, this policy orders a different amount each time. |

In the Order-Point, Order-Quantity (s,Q) model, Q is usually determined using the EOQ. We then only need to determine *s*. *s* is the re-order point and includes expected demand over the lead time plus additional safety stock needed to meet specific service levels. However, there are several different service levels that may be used, each providing a different value for *s*.

*Cycle Service Level (CSL)*

The cycle service level is the service level we expect to see per cycle. If this is set at 90% it means that we expect a stock out to occur in 10% of our order cycles. Once the CSL is specified, we simply have to calculate:

*Cost per Stock Out Event (CSOE)*

If we know the cost per stock out event (B1) we can form our total cost equation (NOTE: we are assuming there is a fixed cost per stock out event that does not depend on lost sales):

This can be solved by taking the derivative and setting it equal to 0. The result is:

|  |  |
| --- | --- |
|  | Note that this is only valid when: |

*Item Fill Rate (IFR)*

The item fill rate is the percentage of demand met routinely from our inventory on hand. It is equal to one minus the expected percentage of demand that we expect to be short. If we think of Q as being the expected demand over an order cycle, we have:

If demand is normally distributed, this will be:

Re-arranging, we find that we need to solve for *k* in:

*Cost per Item Short (CIS)*

If we know the cost per stock out event (cs) we can form our total cost equation as:

This can be solved by taking the derivative and setting it equal to 0. The optimal result is found when:

|  |  |
| --- | --- |
|  | Note that this is only valid when: |

NOTE: *D* should be annual demand (units/year) if we are measuring cost of excess inventory (ce) in terms of $/unit/year.

# Periodic Review Policies

A period review policy means that we only measure and place orders periodically (instead of continuously). We are usually given a review time *R* (example: place orders every 2 weeks) and can only place orders at these points. There are two common types of periodic review policies:

|  |  |
| --- | --- |
| Order-Up-To-Level (R, S)   * Policy: Order S-IP every R time periods. * “Replenishment Cycle System” | This will place an order each time period and order-up-to the target inventory level S. |
| Hybrid (R, s, S)   * Policy: Every R time periods, Order S-IP if IP<=s * General case for many policies | This will place an order each time period only if inventory falls below the re-order point s. When it does, it will place an order that will bring inventory to the target inventory level *S*. |

The Order-Up-To-Level (R,S) policy is very commonly used, and will be studied in this course. The second one will not be examined, but it is an interesting case since it generalizes several other ordering policies:

|  |  |
| --- | --- |
| Ordering Policy | Equivalent (R, s, S) Parameters |
| Order-Up-To-Level (R,S) | (R, s, S) = (R, S, S)  Setting the re-order point equal to the target inventory level so that we always place an order if we are below the target inventory level. |
| Order-Point, Order-Up-To-Level (s,S) | (R, s, S) = (0, s, S)  This continuous-review policy is the same as a periodic review policy with review period (*R*) set to 0 (or approaching zero). |
| Order-Point, Order-Quantity (s,Q) | (R, s, S) = (0, s, s+Q) |
| Base Stock Policy (S) | (R, s, S) = (0, S, S) |

*Order-Up-To-Level (R, S)*

The order-up-to-level policy places an order every *R* period, ordering up to the target inventory level *S*. This is actually similar to the (s, Q) ordering policy, but with a few changes:

|  |  |  |
| --- | --- | --- |
|  | (s,Q) | (R,S) |
| Expected Sales per Order Cycle (Cycle Stock) | Q |  |
| Time that safety stock must cover sales | L | R + L |

Since we will be covering sales over period R+L instead of just L we will also assume that sales are drawn from the normal distribution instead of .

This means that if we solve for *k* in an (s, Q) model with:

The *k* that we obtain will be the optimal k that we want to use in the (R, S) model as well. The only difference is that we will have to convert it back using:

We can also substitute DR for Q and for in the average inventory levels to get:

|  |  |  |
| --- | --- | --- |
|  | (s,Q) | (R,S) |
| Avg. Cycle Stock |  |  |
| Avg. Safety Stock |  |  |

